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Summer Assignment #2

AP Physics I

To be *postmarked* by Monday, July 15th, 2019

Instructions – for full credit:

- Show all work
- Box your final answers
- Make sure to include units in your answer, where appropriate

Name:________________________________

A businessmen will always think in terms of profit. A politician will always think in terms of votes. A real scientist and a real artist is just interested to do.

-Gyorgy Ligeti, Composer (1923-2006)

Summer Assignment #2: Algebra

Section 3: Quadratic Equations

In AP Physics I, we will commonly see quadratic equations – that is, equations of the form

$$ax^2 + bx + c = 0.$$

You have often solved these by factoring before, but it will be rare that we see a factorable equation in physics. If a quadratic equation is not factorable, we will need to use the *quadratic formula*, which says that the two solutions to a quadratic equation are given by the formula

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

Example: Solve the equation

$$2x^2 - 7x + 4 = 3$$

Solution: Before using the quadratic formula, we need to make sure that the right-hand side of the equation is equal to zero. Do this by subtracting 3 from each side:

$$\begin{array}{r} 2x^2 - 7x + 4 = 3 \\ -3 \quad -3 \\ \hline \\ 2x^2 - 7x + 1 = 0 \end{array}$$

Now we can apply the quadratic formula with $a = 2$, $b = -7$, and $c = 1$:

$$x_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{1,2} = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(2)(1)}}{2(2)} = \frac{7 \pm \sqrt{41}}{4}$$

$$x_1 \approx 3.351, \qquad x_2 \approx 0.149 \qquad \blacksquare$$

Problems: Work through the following problems and box your final answers. These will be a part of your Quarter 1 grade. Show all work for full credit.

1. Solve the following quadratic equations.

   (a) $3x^2 - 14x + 8 = 0$

   (b) $4x^2 - 1 = 0$

   (c) $4x^2 + 28x = -49$

   (d) $2x^2 + 5x + 4 = 0$

2. Solve the quadratic equation
$$4.3 + 2.8t - \frac{1}{2}gt^2 = 0$$
for t (in terms of the unknown parameter g).

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3. The height of a projectile (in meters) thrown vertically is given as a function of time (in seconds) by the equation
$$h(t) = 4.00 + 3.50t - 4.91t^2.$$
Determine the time t at which the height $h = 2.00$ meters from the ground. Notice that you will get two possible values for t. Which one do you think is the correct answer?

Section 4: Rational Expressions

Rational expressions are basically fractions containing unknowns (technically, a rational expression is a ratio of two polynomial expressions, but here we will extend the definition to include ratios of other kinds of expressions as well). The procedure for adding and subtracting rational expressions is identical to that of adding and subtracting fractions – rewrite all fractions with a common denominator and add the numarators.

Example: Find the sum

$$\frac{a-b}{ab^2} + \frac{a+b}{a^2b}$$

Solution: First, we need to represent both fractions using a common denominator. By inspection, we see that this is a^2b^2, so we multiply the first term by the fraction a/a and the second term by the fraction b/b:

$$\frac{a-b}{ab^2} \times \frac{a}{a} \qquad + \qquad \frac{a+b}{a^2b} \times \frac{b}{b}$$

$$\frac{(a-b)a}{a^2b^2} + \frac{(a+b)b}{a^2b^2}$$

$$\frac{a^2-ab}{a^2b^2} + \frac{ab+b^2}{a^2b^2}$$

Now that the denominators are the same, we can add the numerators and write the sum over the common denominator a^2b^2:

$$\frac{(a^2-ab)+(ab+b^2)}{a^2b^2}$$

$$\frac{a^2+b^2}{a^2b^2} \qquad \blacksquare$$

Problems: Work through the following problems and box your final answers. These will be a part of your Quarter 1 grade. Show all work for full credit.

1. Combine the following rational expressions and express your answer as a single *simplified* fraction.

   (a) $\dfrac{2}{a} + \dfrac{3}{a-5}$

   (b) $\dfrac{2}{x^2 - 36} - \dfrac{1}{x^2 + 6x}$

   (c) $\dfrac{1}{6x} + \dfrac{2}{3x} - \dfrac{3}{4x}$

2. Combine the following equation for the equivalent resistance for three resistors in parallel (we will learn this near the end of the year)

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

   (a) Re-write the right-hand-side (RHS) of the equation above as a single fraction.
   (b) Using your result from part (a), solve for R_{eq}.

Section 5: Systems of Equations

As far as algebra is concerned, solving systems of linear (and even nonlinear) equations might be *the most important* thing you'll need to be able to do this year! Almost any time we are dealing with 2-dimensional force systems, we will end up with a system of two or three equations. It's perfectly fine if you need some practice right now, but do make sure you know how to do these problems in your sleep by the time school starts in Fall.

In order to get solutions to systems, we must have the same number of unknowns as we have independent equations. Some examples of what this means:

- **Number of equations equals number of unknowns:** The system

$$x + y = 4$$
$$4x - 3y = -1$$

  is solvable since there are two independent equations and two unknowns (x and y).

- **Number of equations is less than number of unknowns:** The system

$$x + y + z = 4$$
$$4x - 3y + 5z = -1$$

  is not solvable since there are two independent equations, but three unknowns (x, y, and z). If this happens in the context of physics, it usually means that we need to go back to the drawing board and derive another equation (from kinematics, forces, energy, momentum, etc.).

- **Number of equations is greater than number of unknowns:** The system

$$x + y = 4$$
$$4x - 3y = -1$$
$$3x - y = 2$$

  is not solvable since there are three independent equations, but only two unknowns (x and y). It is *possible* for there to be some set of solutions that satisfies all three equations, but it is unlikely. We say that the system is *overconstrained*.

- **Two or more equations are redundant:** The system

$$x + y = 4$$
$$3x + 3y = 12$$

  is actually the same equation repeated (the second equation is really just the first equation with each side multiplied by 3), so they are not independent. In reality, we only have one equation with two unknowns, which means we need another equation.

Example: Solve the system of equations

$$\begin{aligned} 7x - 5y &= 18 \\ 5x - 2y &= 15 \end{aligned}$$

for x and y.

<u>Solution:</u> Start by picking one equation and solving for one variable. Since 5 goes into 15 nicely, let's solve for x in the second equation:

$$\begin{aligned} 5x - 2y &= 15 \\ 5x &= 2y + 15 \\ x &= \frac{2}{5}y + 3 \end{aligned}$$

Now that we have "used up" the second equation, let's go to the first equation and replace x with what we found:

$$\begin{aligned} 7x - 5y &= 18 \\ 7\left(\frac{2}{5}y + 3\right) - 5y &= 18 \end{aligned}$$

Now that we only have one equation and one unknown (y), proceed by solving for y:

$$\begin{aligned} 7\left(\frac{2}{5}y + 3\right) - 5y &= 18 \\ \frac{14}{5}y + 21 - 5y &= 18 \\ -\frac{11}{5}y &= -3 \\ y &= \frac{15}{11} \end{aligned}$$

Use this value of y with the equation found above for x to solve for x:

$$\begin{aligned} x &= \frac{2}{5}y + 3 \\ x &= \frac{2}{5}(\frac{15}{11}) + 3 \\ x &= \frac{6}{11} + 3 \\ x &= \frac{39}{11} \end{aligned}$$

Thus, the solution to the system is

$$x = \frac{39}{11} \approx 3.54 \qquad y = \frac{15}{11} \approx 1.36 \qquad \blacksquare$$

Problems: Work through the following problems and box your final answers. These will be a part of your Quarter 1 grade. Show all work for full credit.

1. Solve the following systems of equations. Check your answer by plugging your final results back into the equations.

   (a) $5x + y = 13$
   $3x = 15 - 3y$

   (b) $2x + 4y = 36$
   $10y - 5x = 0$

   (c) $2x - 4y = 12$
   $3x = 21 + 6y$

   (d) $10x + 7y = 49$
   $10y - x = 70$

2. Solve the following system of equations (for x and y) in terms of a and b

$$3ax - by = 10$$
$$2bx + 4ay = 3$$

3. Solve the following systems of equations. Check your answer by plugging your final results back into the equations.

   (a) $x + y + z = 10$
   $3x + 3y = -6$
   $x - 4z = 12$

   (b) $2x - 4y + 3z = -2$
   $3x - y = 0$
   $5x = -15$

4. Suppose that a mechanics problem is solved and results in the system of equations

$$N - mg\cos\varphi = 0$$
$$mgR\sin\varphi + \mu N = I_G\alpha$$

In the problem, suppose that the quantities μ, m, g, R, φ, and I_G are all known, and quantities N and α are unknown. Solve for α in terms of only the known quantities.

5. Suppose that a mechanics problem is solved and results in the system of equations

$$\begin{aligned} -\mu mg + T &= m a_{G,x} \\ -R\mu mg - RT &= I_G \alpha \\ a_{G,x} &= R\alpha \end{aligned}$$

   In the problem, suppose that the quantities μ, m, g, R, and I_G are all known, and quantities T, $a_{G,x}$, and α are all unknown. Solve for $a_{G,x}$ in terms of only the known quantities.