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Summer Assignment #1

AP Physics I

To be *postmarked* by Monday, July 1st, 2019

Instructions – for full credit:

- Show all work
- Box your final answers
- Make sure to include units in your answer, where appropriate

Name:______________________________

What Descartes did was a good step. You have added much several ways, and especially in taking the colours of thin plates into philosophical consideration. If I have seen further it is by standing on the shoulders of Giants.

-Sir Isaac Newton, February 5, 1676 (in a letter to Robert Hooke)

Summer Assignment #1: Trigonometry

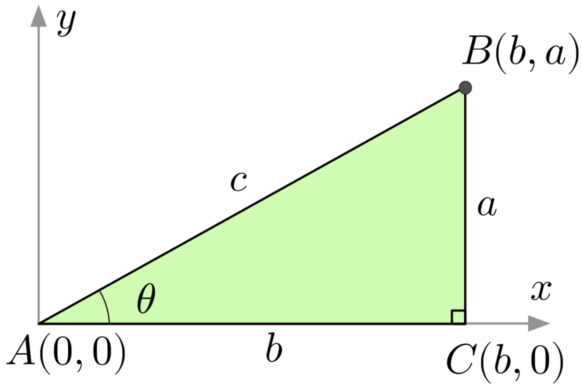
Section 1: Trigonometric Functions

You are familiar with the Pythagorean Theorem, which relates the two legs and the hypotenuse of a right triangle:

$$a^2 + b^2 = c^2$$

You also probably remember the mnemonic device *SOHCAHTOA* from geometry, used to help students remember the three primary trigonometric functions:

- **S**ine = **O**pposite/**H**ypotenuse
- **C**osine = **A**djacent/**H**ypotenuse
- **T**angent = **O**pposite/**A**djacent

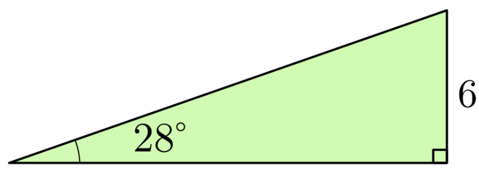


The three functions above take in an angle (in degrees or in radians[1]) and output *ratios of side lengths* of right triangles. They are extremely important when solving physics problems in 2-dimensions (which we will spend a lot of time doing), so you will need to be comfortable using them in your sleep! Mathematically, they are defined as follows:

$$\sin\theta = \frac{a}{c} \qquad \cos\theta = \frac{b}{c} \qquad \tan\theta = \frac{a}{b}$$

Example: Given the right triangle on the right, solve for the side length x.

The first step is to look at what we are given – we have an angle of 28°, an *opposite* side of length 6, and an *adjacent* side of unknown length x. Then, ask yourself – "What trigonometric function relates an angle to an opposite side and an adjacent side?" Referring to **S**OH**C**AH**T**OA, the **T**OA refers to



$$\textbf{T}\text{angent} = \frac{\textbf{O}\text{pposite}}{\textbf{A}\text{djacent}}$$

which is what we need:

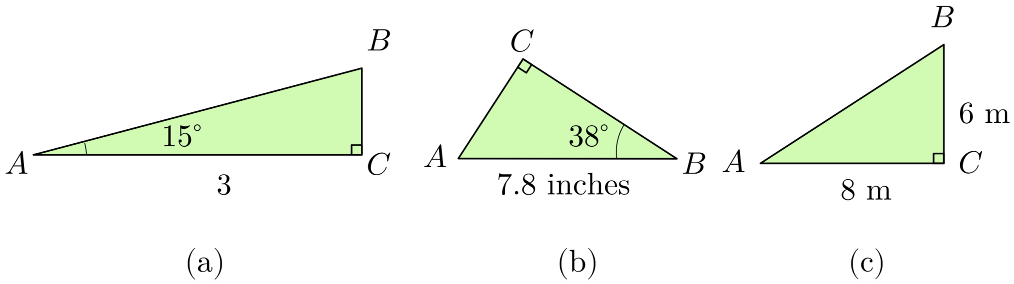
$$\tan(28°) = \frac{\text{opp}}{\text{adj}} \qquad \Rightarrow \tan(28°) = \frac{6}{x} \qquad \Rightarrow x\underbrace{\tan(28°)}_{\approx 0.532} = 6 \qquad \Rightarrow 0.532x = 6$$

$$\Rightarrow x = 11.278 \qquad \blacksquare$$

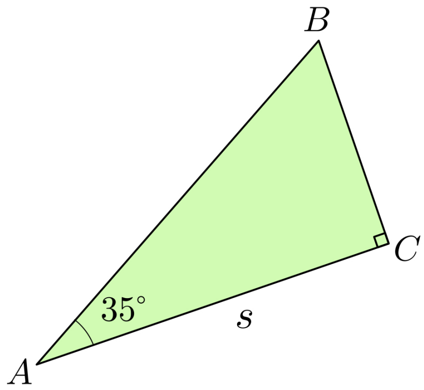
[1]Make sure that your calculator is set to the correct mode!

Problems: Work through the following problems and box your final answers. These will be a part of your Quarter 1 grade. Show all work for full credit.

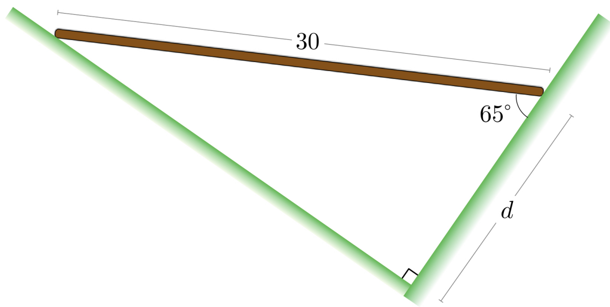
1. Determine all of the unknown lengths for the following three triangles.



2. Determine all side lengths of the triangle on the right as a function of s:



3. The sides of a ravine form a 90° angle at the bottom of the ravine, where they meet. A tree that is 30 meters tall falls so that the angle formed by the ravine's slope and the tree trunk is 65°. How far is the base of the tree from the bottom of the ravine (distance d)?



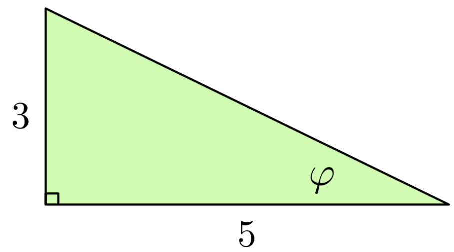
Section 2: Inverse Trigonometric Functions

If you are given side lengths of a right triangle and you need to find its angles, you can use inverse trigonometric functions. Inverse trigonometric functions take in a ratio r of side lengths and output an angle. There are two different notations that are commonly used for inverse functions:

- $\sin^{-1}(r)$, $\cos^{-1}(r)$, and $\tan^{-1}(r)$ (more common)
- $\arcsin(r)$, $\arccos(r)$, and $\arctan(r)$ (kind of outdated, but still shows up occasionally)

Inverse trigonometric functions can usually be accessed on most scientific and graphing calculators by pushing the $\boxed{\text{2nd}}$ key, followed by the key for the trigonometric function. We will almost entirely be using degrees in this class (as opposed to radians), so make sure your calculator is on degree mode.

Example: Solve for the angle φ for the triangle shown on the right.



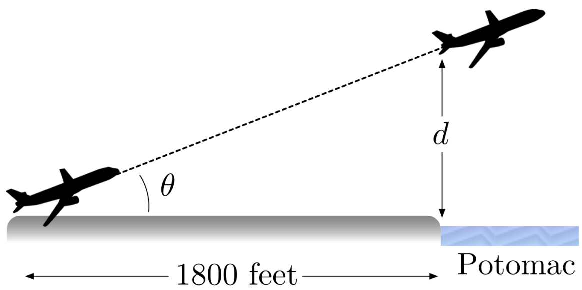
If we imagine standing at angle φ, we notice that we are given the opposite side length and the hypotenuse length. Since OPP/HYP is sine, the angle φ is the angle whose sine is 3/5. Thus we can write

$$\varphi = \sin^{-1}\left(\frac{3}{5}\right)$$

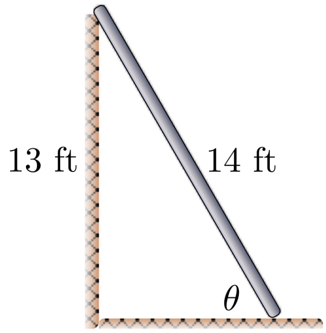
Using a calculator, we find that $\varphi \approx 36.87°$. ■

Problems: Work through the following problems and box your final answers. These will be a part of your Quarter 1 grade. Show all work for full credit.

1. An airplane at Reagan National Airport takes off, with θ representing the flight path angle. If the airplane passes over the bank of the Potomac River 1800 feet away from where takeoff occured and, according to federal regulation, must be 1000 feet in the air by the time it reaches the river, what is the angle of takeoff required? Give your answer rounded to the nearest thousandth.



2. A 14 foot ladder is used to scale a 13 foot wall. At what angle of elevation must the ladder be situated in order to reach the top of the wall?



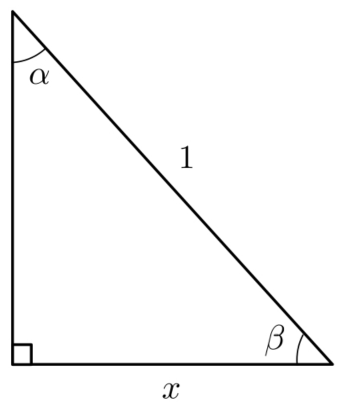
3. A ramp is needed to allow vehicles to climb a 2.0 foot wall. Tha angle of elevation in order for the vehicles to safely go up must be 30° or less, and the longest ramp available is 5.0 feet long.

   (a) Draw a clearly-labeled diagram.

   (b) Can the 5.0-foot ramp be safely used?

4. Find the values of the following trigonometric functions, given the triangle on the right.
   - Hint #1: You will need the length of the unknown side for some of these problems. Simply apply the Pythagorean Theorem to obtain the missing side as a function of x.
   - Hint #2: You were told that inverse trigonometric functions always take in a ratio (i.e. a fraction), so parts (a), (b), and (c) may look confusing. Write each of them as a fraction with a denominator of 1.

(a) $\sin^{-1}(x)$



(b) $\cos^{-1}(\sqrt{1-x^2})$

(c) $\sin\left(\cos^{-1}(x)\right)$

(d) $\sin\left(\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right)$

5. Two cars A and B leave an intersection (point O) of two perpendicular roads, as shown on the diagram. Car A travels with a constant velocity v_A due north, and and car B travels with a constant velocity v_B due east.

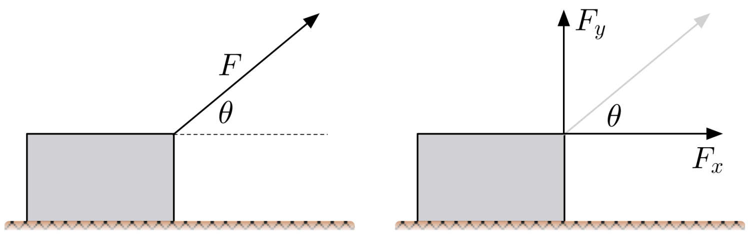
   (a) Draw a clearly-labeled diagram. Make sure to indicate points O, A, and B.

   (b) Determine r_{OA} and r_{OB}, the distances that cars A and B have traveled, respectively. *Hint:* Distance = velocity $\times$ time (for a constant velocity).

   (c) Use your results from part (b) to determine r_{AB}, the distance between the two cars.

   (d) Determine θ, the angle formed by the horizontal road and line AB. Your answer should be in terms of v_A and v_B only.

6. One reason that trigonometry is so important in classical mechanics is that it allows us to represent all forces as combinations of horizontal forces and vertical forces. Consider a block resting on a surface, with a force $F = 10$ Newtons applied at an angle $\theta = 34°$ from the horizontal (shown below on the left).



Original force system Equivalent force system

Using trigonometry, find the combination of forces F_x and F_y that, when applied to the block, will be equivalent to the original system.

Hint: Just draw a right triangle with the three forces (F, F_x, and F_y) represented by the lengths of the sides, and go about solving for the unknown side lengths the same way you did in the first problem (SOHCAHTOA).